Multiple Roll Systems: Residence Times and Dynamic Response

D. F. Benjamin, T. J. Anderson, and L. E. Scriven
Coating Process Fundamentals Program, Dept. of Chemical Engineering and Materials Science
Center for Interfacial Engineering, University of Minnesota, Minneapolis, MN 55455

In roll coating as in other coating processes the coating liquid often suffers changes in properties on the time scales of the coating flow, that is, from fractions of a second upward depending on the amount of recirculation and recycling. The agents of change may be chemical reaction, colloidal aggregation, or evaporation. Hence the mean residence time and the residence time distribution of the liquid are important to designers and operators of coating processes. Here, building on the examination of roll-coating systems by Benjamin et al. (1995), the residence times of liquid coated by representative arrays of multiple rolls in the "forward roll" mode and relatively starved feed condition (neglecting the possibly significant effects of "rolling banks" and other internal recirculations when they are present) are analyzed. The dynamic response of these transfer coaters to step changes in the feed gap and to periodic gap changes, as from roll and bearing run-out, are also analyzed. No reports of operating or laboratory experiments are available for comparison. Nevertheless the results make plain how these quality-limiting features may depend on the number of rolls used; their sizes, speed, and arrangement, and the properties of the coating liquid.

Introduction

Roll coating is a common technique for metering, distributing and applying a continuous liquid film onto a flexible substrate or web. The majority of roll coaters utilize multiple rolls for this purpose. A common type is the forward-roll transfer coater (Booth, 1970; Weiss, 1984). Contemporary transfer coaters typically use a series of three, four, or five counterrotating rolls. Such configurations prevail in "coil coating," the precoating of continuous strips of metal, and in "release liner" coating, that is, the application of ultrathin (< 1 micron wet) silicone release materials on paper and polymer films.

Coating liquids are designed to solidify partially or completely after they are applied. Often their properties are labile, because they contain volatile solvents that can evaporate, reactive components that can polymerize and cross-link, and metastable suspensions of colloidal particles that can flocculate and coagulate. If these transformations proceed appreciably on the time scales of residence in the coating flow, their effects on the flow, through the rheological properties of the liquid, can give rise to unwanted blemishes and unacceptable defects. A spectrum of parallel time scales, that is, any distribution of *transit times through*, or *residence times in* a liquid coating operation, multiplies the danger because it produces nonuniform extent-of-transformation and hence nonuniform rheological properties. Obvious examples are partially gelled slugs and coagulated clots that sometimes form in the centers of recirculating gyres. Residence times in roll-coating systems are inherently distributed.

The longest and most widely spread segments of residence times in roll-coating systems arise in the commonly used flow distribution devices: the pool in an open feed pan to which the excess liquid lifted out by the pick-up roll is returned; and the pond between gate rolls (or between a blade and roll) to which liquid is fed on demand (Benjamin and Scriven, 1992). These devices are essentially partially stirred tanks, which have been treated in the literature on mixing. For that reason, and because pools (and ponds, too) tend to be replaced by short transit-time chamber-and-slot devices as quality requirements rise, what are considered here are roll-coating systems that are operated with a pond between gate rolls or with a chamber-and-slot delivery to the first roll, which is the end-roll or pick-up roll.

Current address of D. F. Benjamin: Consolidated Papers, Inc., Wisconsin Rapids, WI 54495.

Another contributor to residence times is the "rolling bank" or body of liquid (with one or more recirculating gyres) that may be allowed to build up on the inflow side of a gap or nip downstream of the ponded gap or delivery position. For similar reasons, only systems that are operated with relatively starved gaps or nips are considered here: the possibly significant effects of "rolling banks" and other internal recirculations are neglected. By gap is meant, strictly, a clearance between rolls that would be positive were they to be stopped; by nip, a clearance that would be negative. Hereafter, *gap* is used as the generic term for both circumstances.

The remaining major contributors to residence times are the liquid hold-up as films on the multiple rolls of roll coating system, and the recycling of portions of that liquid by virtue of film-splits on the outflow side of forward-roll gaps (or film wipes from the output side of reverse-roll gaps, not considered here). These films and film splits are the starting point of the analysis that follows.

Multiple roll systems are inherently unsteady due to unavoidable imperfections in rolls, shafts, bearings, and drives as well as misalignment of these roll-coater components. This unsteadiness results in ever-changing gaps and nip widths that can give rise to variations of coated film thickness in the down-web direction. The unsteady flow may affect mixing in the gaps and hence the residence time distributions. How sensitive the coated film thickness is to such ongoing disturbances and to intentional adjustments of operating conditions is also in the following.

Background

In a previous article (Benjamin et al., 1995) forward-roll, or transfer, coaters and other multiple-roll configurations were examined. The results of that work are equations that relate coated layer thickness, or "film thickness," to the operating parameters and the liquid properties. Figure 1 depicts a system of $n+1$ counterrotating rolls in a forward-roll transfer coater. The intermediate rolls (roll 2 through roll n) each carry two films: the primary film h_i, which flows *toward* the application gap (gap_1), and the recycle film h'_i, which flows *back* toward the feed gap (gap_n). As reported in the previous article, the equation for the coated film thickness h_1 from a pond-fed, undoctored transfer coater with $n+1$ rolls is

$$h_1 = \frac{\lambda_n g_n (1+V_{n+1,n}) V_{n,1}}{2\left[1+\alpha_n (V_{n+1,n})^{1+\beta_n}\right]\left[1+\sum_{i=1}^{n-1}\prod_{j=n-i}^{n-1}\alpha_j (V_{j+1,j})^{1+\beta_j}\right]}, \tag{1}$$

where g_n is the feed gap width between the two end rolls (roll $n+1$ and roll n); λ_n is the dimensionless flow rate through the feed gap; $V_{i,j}$ represents the ratio of the speed of roll i (V_i) to the speed of roll j (V_j), that is, $V_{i,j} = V_i/V_j$; and α_i and β_i are the parameters of the film-split correlation of each gap flow:

$$\frac{h'_{i+1}}{h_i} = \alpha_i (V_{i+1,i})^{\beta_i} \qquad 1 \le i \le n. \tag{2}$$

The film-split parameters, α_i and β_i depend on the relative magnitude and direction of the force of gravity, inertia, and net viscous stress in the film-splitting flow. Generally, α_i varies between 0.6 and 1.5; β_i, between 0.3 and 0.67 (Hintermair and White, 1965; Benkreira et al., 1981, 1993; Savage, 1982; Coyle et al., 1986; Benjamin et al., 1993).

The thicknesses of the intermediate films are related to the coated film thickness through the roll speed ratios and the film-split parameters:

$$h_i = \frac{h_1}{V_{i,1}} + h'_i \qquad 2 \le i \le n, \tag{3}$$

$$h'_i = h_{i-1}\alpha_{i-1}(V_{i,i-1})^{\beta_{i-1}} \qquad 2 \le i \le n \tag{4}$$

$$h_{n+1} = h_n \alpha_n (V_{n+1,n})^{\beta_n}. \tag{5}$$

The films h_i and h'_i are the film flow rate per unit width Q_i divided by the roll speed V_i: $h_i \equiv Q_i/V_i$, and similarly in the case of h'_i. When gravity and roll curvature are negligible, as they often are, the films quickly develop to plug flow; h_i and h'_i are the thicknesses of the developed films then.

The foregoing equations of film thicknesses pertain to steady-state operation. Here, the residence times of liquid in multiple roll coaters, and their response not only to step changes in process conditions but also to continuous sinusoidal perturbation by roll run-out are examined. Several examples of transfer coaters that either appear in the literature or are described in personal communications are used to illustrate the behavior of multiple roll coaters (Booth, 1970; Hebels, 1990; Potjer, 1992; Satas, 1984; Weiss, 1977; Zahn, 1993). These examples are listed in Table 1, which shows roll speed ratios and a summary of the results from the analysis that follows.

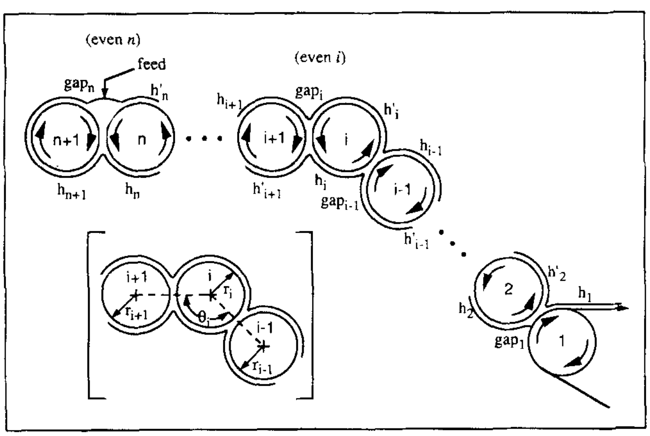

Figure 1. Pond-fed, forward roll transfer coater with $n+1$ counter-rotating rolls forming n gaps.

A roll i can carry two films of liquid: a *primary* film h_i moving *forward*, that is, toward the gap where liquid is applied to the substrate, or web (gap_1); and a *recycle* film h'_i moving *backward*, that is, away from the coating gap and toward the feed gap (gap_n).

Residence Times

Transit times

The source of residence time considered here is the time it takes liquid to travel from one gap to the next on a moving roll surface: *transit time*. A roll i can carry two films of liquid. One is a *primary* film h_i moving *forward*, which means toward the gap where liquid is applied to the substrate, or web. The other is a *recycle* film h'_i moving *backward*, or away from the coating gap. The two transit times depend on the radius r_i and the angular speed ω_i (or the surface speed $V_i = 2\pi r_i \omega_i$) of the roll i on which the film is carried, as well as the angle θ_i between the two gaps. The transit time t_i for the primary film h_i to travel forward from gap$_i$ to gap$_{i-1}$, and the transit time t'_i for recycle film h'_i to be carried backward from gap$_{i-1}$ to gap$_i$ are (the lengths of the "coating beads" in the gaps are taken to be negligible compared to the roll perimeter)

$$t_i = \frac{\theta_i}{2\pi\omega_i} = \frac{\theta_i r_i}{V_i} \qquad 1 \le i \le n+1, \qquad (6)$$

$$t'_i = \frac{2\pi - \theta_i}{2\pi\omega_i} = \frac{(2\pi - \theta_i) r_i}{V_i} \qquad 2 \le i \le n. \qquad (7)$$

The angle, θ_i (in radians), is located between the center of roll i and the centers of its two neighboring rolls (roll $i-1$ and roll $i+1$) measured in the forward sense, that is, on the side of film h_i. Below, θ_i is called the coating angle. Because the end-roll (roll $n+1$) has only one neighbor (roll n), the angle associated with it, θ_{n+1}, is 2π, and so the transit time t_{n+1} of film h_{n+1} is simply $1/\omega_{n+1}$. The backing roll (roll 1) carries the web into gap$_1$ to be coated, and therefore does not carry a liquid film, but a transit time associated with roll 1 is defined and used as the characteristic time of the system: $t_1 \equiv 1/\omega_1$ in time per radian, that is, the period of roll 1 divided by 2π.

Laminar flow, no mixing in gaps

When the flow between and around rolls is steady and laminar and the diffusion of critical components of the liquid across streamlines is negligible, the recycled liquid does not mix with the freshly arriving liquid. Figure 2 depicts patterns of dividing streamlines in laminar flow in five-roll coaters, ordinary and doctored. In both types there is one path, or streamsheet, that begins as the feed stream, travels through the primary film flows, and is coated. This liquid travels through a system of $n+1$ rolls in time t_{min}, which is the sum of the transit times of the primary films and is the minimum residence in the system:

$$t_{min} = \sum_{i=2}^{n} t_i. \qquad (8)$$

Figure 3 shows this minimum residence time, in units of t_1, of two-, three-, four- and five-roll coaters with roll speeds arranged in an arithmetic progression (Figure 3a) and in a geometric progression (Figure 3b) from the end-roll speed to the line speed. The rolls have equal roll radii and are arranged in a straight line ($r_{i,1} = 1$, $i = 2, n+1$; $\theta_i = \pi, i = 2, n$).

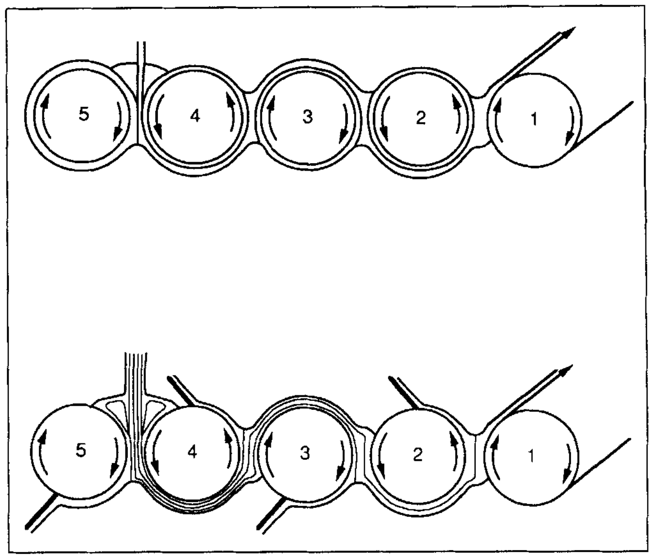

Figure 2. When the flow through a multiple roll coater is laminar and steady, a streamsheet begins as the feed stream and is coated; the rest is either trapped indefinitely in gyres or in recirculating flows around the rolls in the ordinary transfer coater (top) or removed in the doctored transfer coater (bottom).

As the end-roll speed falls, t_{min} rises moderately when the progression is arithmetic, sharply when it is geometric. As the end-roll speed falls to zero in the geometric progression, all roll speeds approach zero and t_{min} approaches infinity. The logarithmic scale in Figure 3b illustrates better the sharp rise of t_{min} as the end-roll speed falls to zero when the roll speeds are in a geometric progression, that is, the ratios of speeds of successive rolls are all the same. Values of t_{min} for the six industrial examples are listed in Table 1.

If the flow were truly laminar, steady, and two-dimensional, the liquid contained in the recirculating flows carried by the roll surfaces (and in the gyres of ponds and rolling banks) would be trapped indefinitely in ordinary transfer coaters. That liquid's residence time would be very long—some would have resided since startup, some since the last change in roll speeds. In reality the flow through a system of gaps is not strictly steady and two-dimensional because there is run-out and at least slight misalignment of the rolls. Consequently, there must be some discharge and replacement of liquid in the recirculating flows, that is, some mixing of recycle with fresh liquid. There must also be some diffusional interchange if the recycle changes in composition, and this would be enhanced by the extending and shearing of liquid in the gaps. Mixing might be facilitated by reciprocating translation in the axial direction by one or more rolls, as is done in forward-roll systems of inking trains on printing presses. It is also reasonable to expect the mixing to be more complete in a gap where one of the rolls is soft and the clearance narrow, that is, the films thin, especially when the substrate is also rough. Then the substrate may actually (slowly) clean the rolls.

A way to avoid the very long residence time of liquid carried next to the roll surfaces is to doctor them, as indicated in

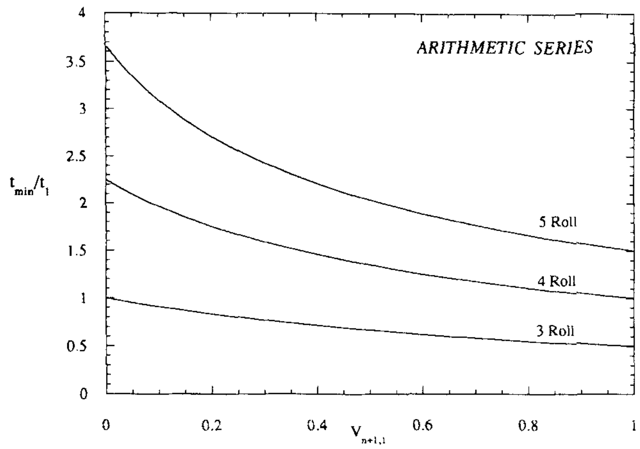

Figure 3a. Minimum time for liquid to pass through three-, four- and five-roll coaters as a function of the ratio of end-roll speed to line speed.

The intermediate roll speeds are arranged in an arithmetic progression; $\theta_i = \pi (i = 2, n)$; $r_{i,1} = 1$ $(i = 2, n+1)$.

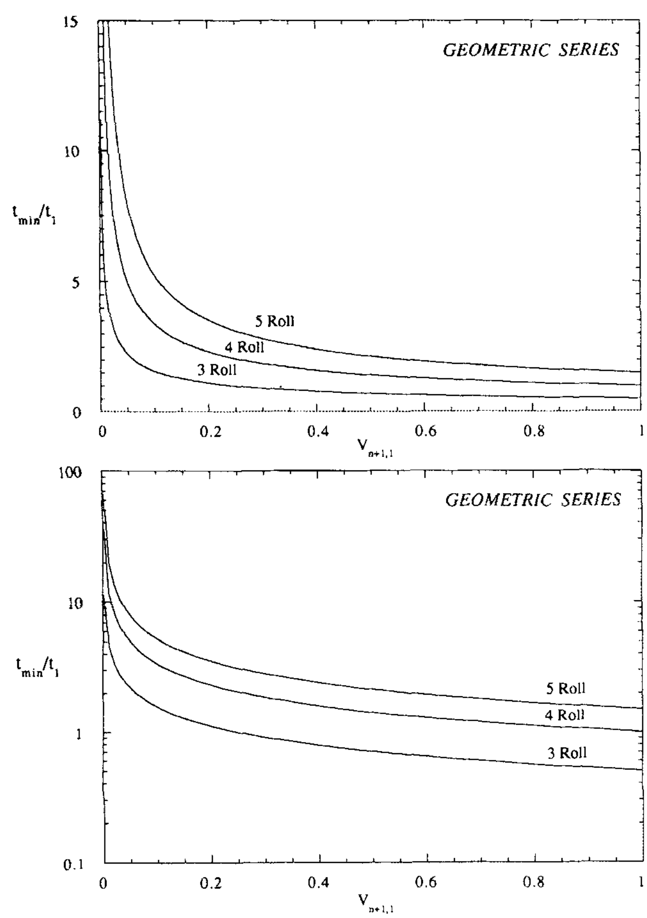

Figure 3b. Minimum time for liquid to pass through three-, four-, and five-roll coaters as a function of the ratio of end-roll speed to line speed.

The intermediate roll speeds are arranged in geometric progression; $\theta_i = \pi$ $(i = 2, n)$; $r_{i,1} = 1$ $(i = 2, n+1)$; the bottom plot has a logarithmic scale.

Figure 2b. In the doctored transfer coater the liquid in the recycle streams is removed by a scraper, or doctor blade, on each roll (alternatively, on some rolls); it may or may not be returned to the feed system for remixing.

Perfect mixing in gaps

Estimating how much mixing and diffusion take place in the gap flows is difficult. The extreme of none is surely unrealistic. The opposite extreme of perfect mixing seems unlikely but probably a better approximation to reality. When all the liquid in a gap is well mixed, the probability of any tiny parcel of liquid departing on a given roll is independent of which roll the particle entered on and is determined by the relative flow rates leaving on each roll. If the gaps are taken to be mixers with neither holding volume nor holdup time, only transit times contribute to residence time. Liquid leaving gap$_i$ next acquires either residence time t_i in transit to gap$_{i-1}$ or residence time t'_{i+1} in transit to gap$_{i+1}$, according to which film, primary or recycle, it exits with. All of the liquid in a given film acquires the same amount of transit time flowing to the next gap and thus the average age of liquid in that film increases by that transit time, regardless of the distribution of residence times in the film. By defining an average age of the liquid leaving each gap and relating these ages to the transit times and flow rates between each pair of gaps, the average age, or average residence time, in the coated film can be found. Although recycle streams bring liquid of different ages to a gap, the average age of liquid leaving any gap flow remains constant (once the initial transient has decayed away).

Average Residence Time. In the limit that the streams entering a gap become perfectly mixed within the gap flow and acquire no additional residence time therein, an age balance based on the average ages of the inlet and outlet streams at gap$_i$ can be written:

$$Q_{i+1}(a_{i+1} + t_{i+1}) + Q'_i(a_{i-1} + t_i) = (Q_i + Q'_{i+1})a_i \qquad 2 \le i \le n-1. \quad (9)$$

Here, a_i is the average age of the liquid exiting gap$_i$; Q_i is the flow rate of film h_i $(Q_i = V_i h_i)$; and t_i is the transit time of film h_i (t_1 is defined to be $1/\omega_1$). The age balance at gap$_1$ is different because there is only one entering film:

$$Q_2(a_2 + t_2) = (Q_1 + Q'_2)a_1. \quad (10)$$

Here, a_1 is the average age of the liquid exiting gap$_1$, and a_2 is the average age of liquid exiting gap$_2$. The age balance at the feed gap (gap$_n$) is also different because the feed stream enters here and a holdup time in the feed pond must be accounted for:

$$Q_{n+1}(a_n + t_{n+1} + t_{\text{pond}}) + Q'_n(a_{n-1} + t'_n + t_{\text{pond}}) + Q_1(a_0 + t_{\text{pond}}) = (Q_{n+1} + Q_n)a_n. \quad (11)$$

Here, a_0 is the age of the liquid in the feed stream and t_{pond} is the holdup time associated with the feed pond volume. The flow rate of the feed stream is taken to be equal to the rate at which liquid is being coated, Q_1. Solving Eqs. 9 through 11

Table 1. Six Examples of Transfer Roll Coaters Taken from Literature Sources and Personal Communications*

	3-Roll			4-Roll			5-Roll			
	Booth 1970	Hebels 1990	Equal Speed	Satas 1984	Zahn 1993	Equal Speed	Potjer 1992	Weiss 1977	Equal Speed	
$V_{1,1}$	1	1	1	1	1	1	1	1	1	
$V_{2,1}$	1	0.8	1	1	1.5	1	1.05	1	1	$\theta_i = \pi$
$V_{3,1}$	0.067	0.2	1	0.25	0.75	1	0.85	0.8	1	$r_{i,1} = 1$
$V_{4,1}$				0.1	0.4	1	0.1	0.35	1	
$V_{5,1}$							0.03	0.35	1	
$h_1/\lambda_n g_n$	0.264	0.269	0.250	0.120	0.220	0.167	0.054	0.109	0.125	
t_{min}/t_1	0.5	0.625	0.5	2.5	1	1	6.06	2.55	1.5	
t_{avg}/t_1	1.83	2.33	3.5	6.90	5.25	7	14.3	11.6	11.5	$\alpha = 1$
t_{avg}/t_{min}	3.65	3.73	7	2.76	5.25	7	2.36	4.55	7.67	$\beta = 2/3$
σ	2.74	2.89	3.48	6.8	4.7	6.29	14.8	9.07	9.98	
t_{resp}/t_1	t_{min}/t_1	t_{min}/t_1	t_{min}/t_1	7.5	5	4	20	13.1	7.5	
$\mathfrak{D}$	1	1	1	0.57	0.34	1	0.86	0.25	1	
$h_1/\lambda_n g_n$	0.26	0.248	0.25	0.103	0.19	0.167	0.047	0.096	0.125	
t_{min}/t_1	0.5	0.625	0.5	2.5	1	1	6.06	2.55	1.5	
t_{avg}/t_1	2.31	2.93	3.5	8.64	5.83	7	18.1	13.0	11.5	$\alpha = 1$
t_{avg}/t_{min}	4.62	4.69	7	3.46	5.83	7	2.99	5.1	7.67	$\beta = 1/3$
σ	4.04	3.7	3.48	8.77	5.39	6.29	19.2	10.6	9.98	
t_{resp}/t_1	t_{min}/t_1	t_{min}/t_1	t_{min}/t_1	7.5	4.33	4	19.5	12.6	7.5	
$\mathfrak{D}$	1	1	1	0.59	0.38	1	0.87	0.26	1	

*The roll speeds are from the sources. The values of the coated film thickness $h_1/\lambda_n g_n$, minimum residence time (t_{min}/t_1), average residence time t_{avg}/t_1, standard deviation of residence time σ, dynamic response time t_{resp}/t_1, and run-out damping factor $\mathfrak{D}$ in each of the cases were computed in ways explained in the text ($t_1 \equiv 1/\omega_1$). The film-splitting constant α was chosen to be unity in every case; two values of the film-splitting exponent β were examined as tabulated.

for coaters with just a few rolls ($n = 1$, 2, 3, and 4) and using induction leads to an equation for the average age, t_{avg}, of liquid coated by a transfer coater with $n+1$ rolls:

$$t_{avg} = a_1 = (a_0 + t_{pond}) + \frac{1}{Q_1}\left[\sum_{i=2}^{n}(Q_i t_i + Q_i' t_i') + Q_n'(t_{pond}) + Q_{n+1}(t_{n+1} + t_{pond})\right]. \quad (12)$$

This can be rewritten in terms of the film thicknesses, roll speeds, roll radii, and roll alignment angles:

$$t_{avg} = a_1 = (a_0 + t_{pond}) + \frac{1}{V_1 h_1}\left[\sum_{i=2}^{n} r_i(\theta_i h_i + \theta_i' h_i') + 2\pi r_{n+1} h_{n+1} + t_{pond}(V_n h_n' + V_{n+1} h_{n+1})\right]. \quad (13)$$

Figure 4 shows the average residence time, $t_{avg} = a_1$, in units of $t_1 \equiv 1/\omega_1$, of liquid coated by two-, three-, four-, and five-roll coaters when (1) the pond volume is negligible ($t_{pond} = 0$, $a_0 = 0$); (2) the rolls have equal radii and are arranged in a straight line ($r_{i,1} = 1$, $i = 2, n+1$; $\theta_i = \pi$, $i = 2, n$); and (3) the roll speed ratios are in arithmetic progression (Figure 4a) or geometric progression (Figure 4b) from the end-roll to line speed.

Predictions are shown for two values of the film-splitting exponent: $\beta = 1/3$ and $\beta = 2/3$. The lower the film-splitting exponent, the greater the proportion of liquid that flows into the recycle films, because they are carried away by the slower moving roll at each gap, and hence the longer the average residence time.

As the end-roll speed falls, all of the intermediate roll speeds fall and the transit times rise accordingly; the effect is to increase the average residence time. Yet, as speed ratios between adjacent rolls rise, $V_{i+1,i}$ rise, less liquid flows into the recycle streams; the effect is to decrease the average residence time. This tradeoff results in a local maximum in average residence time at $V_{n+1,1} \approx 0.125$ for a five-roll, $V_{n+1,1} \approx 0.2$ for a four-roll, and $V_{n+1,1} \approx 0.55$ for a three-roll system when the speeds are arranged in an arithmetic series and $\beta = 1/3$.

In many pond-fed transfer coaters, parcels of liquid can spend a long time in the rolling bank flow of the pond, especially if the bank of liquid is large and contains recirculations or gyres as are often present in practice and have been documented (Benjamin, 1994). If the duration of time spent in the pond flow t_{pond} can be determined or estimated, this contribution to the total residence time can be accounted for with Eq. 13. This equation predicts a linear relationship between the mean residence time t_{avg} and the pond residence time t_{pond} for a given number of rolls and roll speed ratios. The linear dependence is depicted, for the situation when the roll speeds are equal, in the top plot of Figure 5. The lower plot of Figure 5 shows how the dimensionless mean residence time t_{avg}/t_1 depends on the number of rolls in the system at four different pond residence times t_{pond}/t_1 when all the roll speeds are equal.

When the average residence time is plotted in units of t_{min}, as in Figure 6, the three-, four-, and five-roll coaters are quite similar. At equal roll speeds, liquid in the three- and four-roll coaters takes, on average, seven times longer to flow through the roll system than the quickest possible path through the

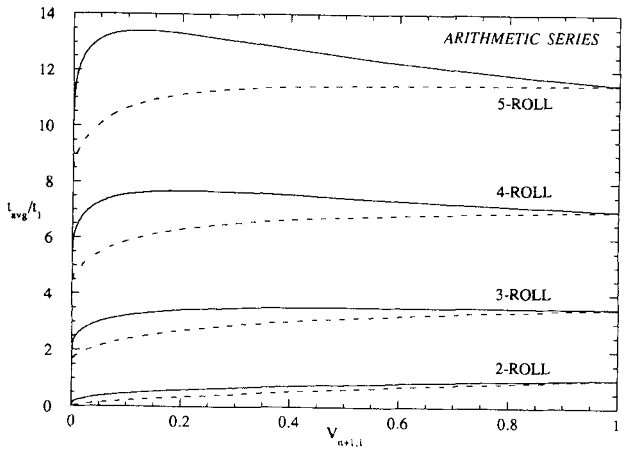

Figure 4a. Average residence time of the coated liquid in units of t_1, roll 1's period divided by 2π, as a function of the ratio of end-roll speed to line speed $V_{n+1,1}$ and the film-splitting exponent β.

The roll speeds are arranged in arithmetic progression from the end-roll speed to line speed; $\alpha = 1$; $t_{\text{pond}} = 0$; $\theta_i = \pi$ $(i = 2, n)$; $r_{i,1} = 1$ $(i = 2, n+1)$: — $\beta = 1/3$; --- $\beta = 2/3$.

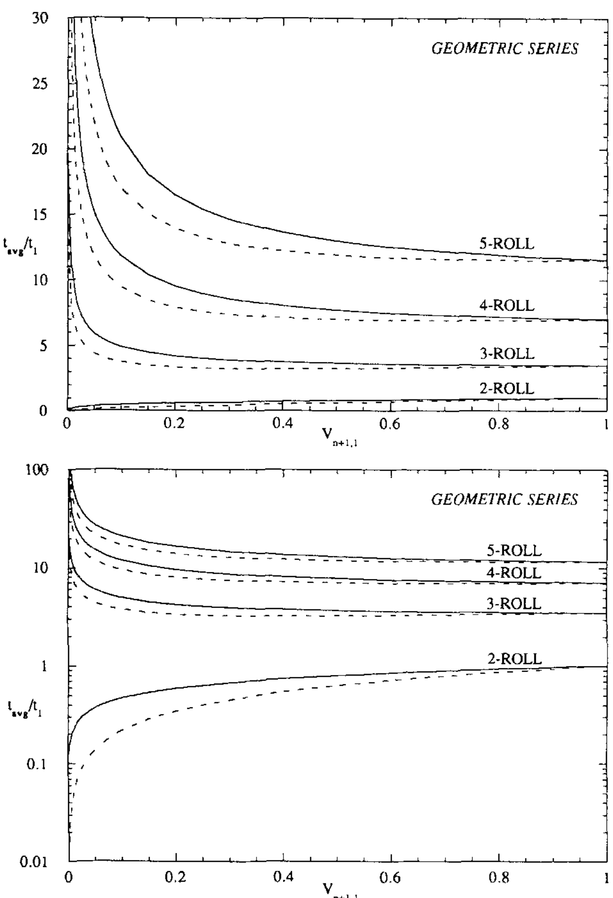

Figure 4b. Same as Figure 4a except that the roll speeds are arranged in geometric progression from end-roll speed to line speed.

The lower graph has a logarithmic scale: — $\beta = 1/3$; --- $\beta = 2/3$.

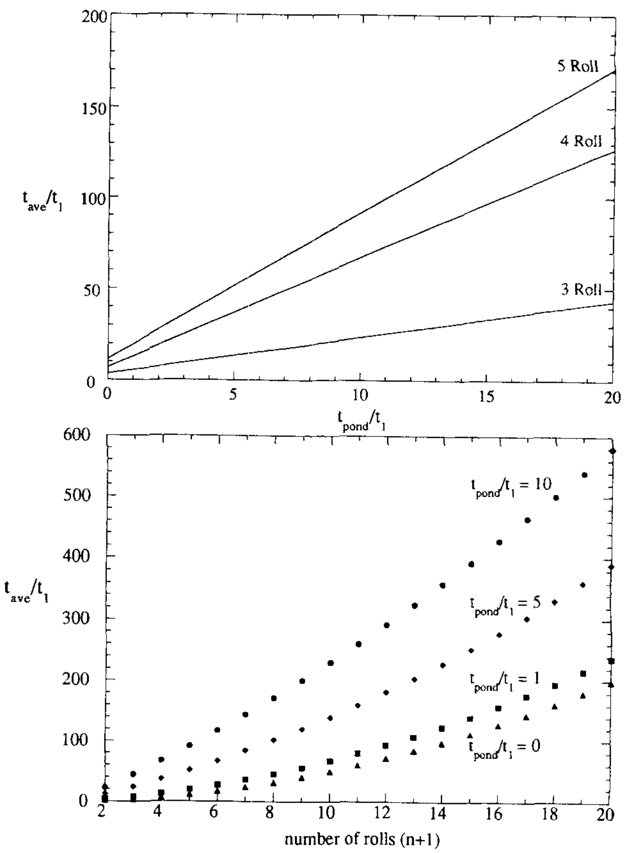

Figure 5. Residence time depends on how long the liquid remains trapped in the feed pond.

(Top) The relation between the average residence time t_{avg} and the pond residence time t_{pond} for a given number of rolls and equal roll speeds. (Bottom) The dependence of residence time on the number of rolls in the system at four different values of t_{pond}.

rolls (no recycling): $t_{\text{avg}}/t_{\text{min}} = 7$. Residence time in the five-roll coater is slightly longer: $t_{\text{avg}}/t_{\text{min}} = 7.67$ (t_{min} is defined at Eq. 8). In the cases shown in Figure 6, there is little difference between the arithmetic and the geometric progressions of intermediate roll speeds (at the same value of film-splitting exponent); however, the dependence of $t_{\text{avg}}/t_{\text{min}}$ on the film-splitting exponent is noteworthy: the lower the exponent, the longer the average residence time.

The minimum and average residence times in the industrial examples of roll-coating systems examined here are listed in Table 1. The values of $t_{\text{avg}}/t_{\text{min}}$ in these examples turn out to be close to those that can be read, for the same number of rolls and the same end-roll speed, from Figure 6, which pertains to the geometric and arithmetic progressions. Thus the ratio $t_{\text{avg}}/t_{\text{min}}$ is relatively insensitive to the progression of intermediate roll speeds.

Average age of the liquid coating reflects the extent of recycling within the system. It may be useful for estimating the likelihood of difficulties with reacting or colloidal liquids or volatile solvents. However, difficulties are more likely to develop the greater the age disparity between older and younger parcels of liquid in the system. Indeed, the residence time distribution is generally more important than the mean residence time.

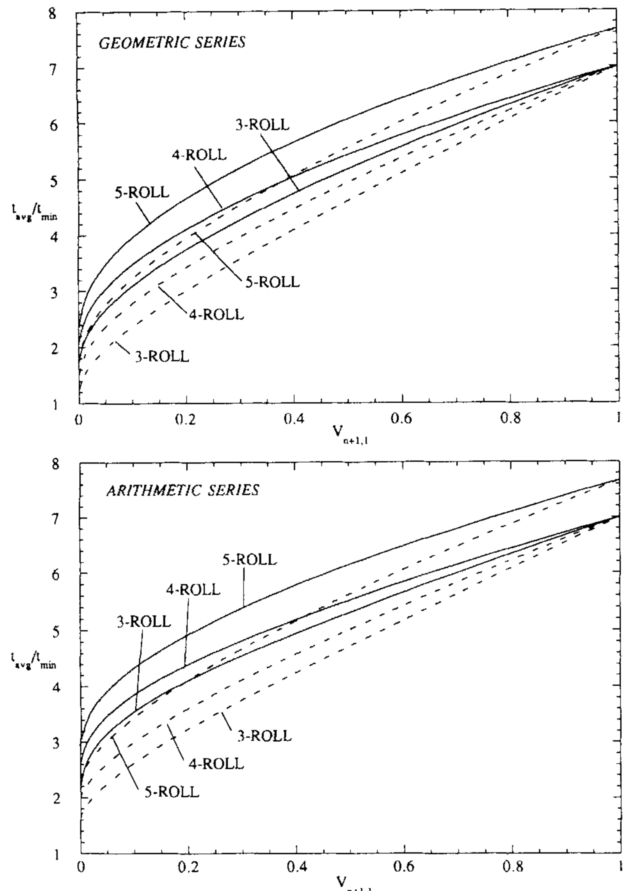

Figure 6. Average residence time in the coated liquid in units of t_{min} the minimum residence time from Eq. 8, as a function of the ratio of the end-roll speed to line speed $V_{n+1,1}$ and the film-splitting exponent β.

The roll speeds are arranged in geometric progression (top) and arithmetic progression (bottom) from the end-roll speed to line speed: $\alpha = 1$; $t_{pond} = 0$; $\theta_i = \pi$ ($i = 2, n$); $r_{i,1} = 1$ ($i = 2, n+1$): — $\beta = 1/3$; --- $\beta = 2/3$.

Markov Chain Model of Multiple Roll System. With perfect mixing in the gaps, a transfer coater is a stochastic process because the location of a liquid parcel is determined by random events. At each gap, as idealized here, a given parcel moving through the system exits in the primary film or the recycle film totally at random. The path of a parcel can be thought of as a random walk through the gaps until it leaves gap_1 in the primary film and is coated. The random variable is the number of the gap. Because the probabilities of which gap the parcel moves to next depend solely on the gap it is in currently and not on those it has previously visited, the stochastic process is a Markov one (Feller, 1968). And because the number of possible "states" the particle can be in —the number of gaps plus one (the "state" of being coated) —is finite, the transfer roll coater can be modeled as a Markov chain. In a roll coater with $n+1$ rolls, a liquid parcel has $n+1$ possible states: s_0, the state of being coated; s_1, in gap_1; s_2, in gap_2; . . . ; s_n, in gap_n. The state of being in gap_n (s_n) is a reflecting state (or reflecting barrier) because liquid that leaves gap_n in the recycle stream returns to gap_n. The state of being coated (s_0) is an absorbing state because liquid that leaves gap_1 in the primary stream—liquid that is coated —remains coated, never to return. This Markov chain model of the transfer roll coater is said to be stationary or have stationary transition probabilities (also called a homogeneous Markov chain) because the transition probabilities (probabilities of flowing from one gap to another) are constant, that is, independent of how many gaps the particle has visited already.

The transition probabilities generally differ from gap to gap because they are set by the relative flow rates of the primary and recycle streams leaving the gap, which in turn are determined by the roll speeds and the film-splitting parameters at each gap. If p_{ij} is the probability of moving from gap_i to gap_j, the transition probabilities for a transfer roll coater with $n+1$ rolls are

$$p_{00} = 1$$

$$p_{0j} = 0 \qquad j = 1, \ldots, n$$

$$p_{ij} = \begin{cases} 0 & j < i-1 \\ \dfrac{Q_i}{Q_i + Q'_{i+1}} = \dfrac{1}{1 + \alpha_i (V_{i+1,i})^{1+\beta_i}} & j = i-1 \\ 0 & j < i-1 \\ \dfrac{Q_i}{Q_i + Q'_{i+1}} = \dfrac{\alpha_i (V_{i+1,i})^{1+\beta_i}}{1 + \alpha_i (V_{i+1,i})^{1+\beta_i}} & j = i-1 \\ 0 & j < i-1 \end{cases}$$

$$i = 1, \ldots, n-1; \quad j = 0, \ldots, n$$

$$p_{nj} = 0 \qquad j = 0, \ldots, n-2$$

$$p_{nn-1} = \frac{Q_n}{Q_n + Q'_{n+1}} = \frac{1}{1 + \alpha_n (V_{n+1,n})^{1+\beta_n}}$$

$$p_{nn} = \frac{Q_n}{Q_n + Q'_{n+1}} = \frac{\alpha_n (V_{i+1,i})^{1+\beta_i}}{1 + \alpha_n (V_{n+1,n})^{1+\beta_n}}. \tag{14}$$

When the rolls have equal speeds and α is one, all the gap-to-gap transition probabilities (except those from the state of being coated) are one-half.

Considerable mathematical theory has been developed for stationary Markov chains (Feller, 1968; Chung, 1967, 1979), and some of its results relate directly to the situation at hand. One result states that the probability of a parcel leaving the system before infinite time is one. Hence no parcel can accrue infinite residence time in a perfectly mixed transfer roll coater. Another result leads to the expected residence time of a parcel starting in the feed gap and making n gap movements (Howard, 1969). The asymptotic behavior of the expected residence time as n grows large converges to the same result given by Eq. 13; yet no information about the *distribution* of residence times can be gleaned from this. Except in the special case of rolls all sharing equal angular velocity and arranged in a straight line, the transit times between gaps are all different. This significantly complicates the Markov chain model and makes developed and published theory of Markov chains difficult to apply. Instead of pursuing this further, it seems more effective to simulate the transfer roll coater as a

Markov chain by a Monte Carlo method on a computer, which can be done fairly simply.

Monte Carlo Simulation of the Markov Chain Model. The rudiments of Monte Carlo methods are described by Hammersley and Handscomb (1964), Morgan (1984), and others and are not detailed here. In the present case a parcel is introduced to the system at the feed gap (gap_n). Whether the parcel then flows to gap_{n-1} or stays at gap_n (cycles around roll $n+1$) is decided by picking a random number r (between zero and one) from a uniform distribution and comparing it to the probability of a $\text{gap}_n \rightarrow \text{gap}_n$ transition p_{nn} (or equivalently, to the probability of a $\text{gap}_n \rightarrow \text{gap}_{n-1}$ transition p_{nn-1}), cf. Eq. 14:

$r < p_{nn}$:	recycle to gap_n	$r < p_{ii-1}$:	recycle to gap_{i-1}
$r > p_{nn}$:	advance to gap_{n-1}	$r > p_{ii-1}$:	advance to gap_{i+1}
$r = p_{nn}$:	pick another	$r = p_{nn}$:	pick another.

Whether a parcel at an intermediate gap (gap_i $i = 1, n-1$), moves next to gap_{i+1} or to gap_{i-1} is decided by comparing a random number to the $\text{gap}_i \rightarrow \text{gap}_{i-1}$ transition probability p_{ii-1} (or $\text{gap}_i \rightarrow \text{gap}_{i+1}$ transition probability p_{ii+1}) given by Eq. 14.

This decision-making process is continued until the parcel leaves gap_1 in the primary stream and is coated, that is, a $\text{gap}_1 \rightarrow \text{gap}_0$ transition, all the while keeping track of the total residence time by accumulating the successive transit times of each gap-to-gap movement. In Monte Carlo fashion, the simulation was repeated 10,000 to 100,000 times in order to construct a representative sample of residence times in a given roll-coating system. From this, a residence time distribution was assembled and its mean and standard deviation were computed. In every case examined, the mean was within 1% of the average predicted by Eq. 13.

Residence Time Distributions. The results of these Monte Carlo simulations are shown in Figures 7 and 8. Figure 7 shows the residence time distributions in liquid coated by three-, four- and five-roll transfer coaters in which *all of the* rolls have equal speed ($V_{i,1} = 1$, $i = 1, n+1$). The distributions broaden as rolls are added to the system. At five rolls a bimodal distribution appears. As rolls are added, the mean residence time of course rises. Interestingly, the standard deviation of residence times in every case is only a little less than the mean residence time: the disparity between longer and shorter residence times grows in proportion to the mean residence time as the number of rolls grows. This is a poten-

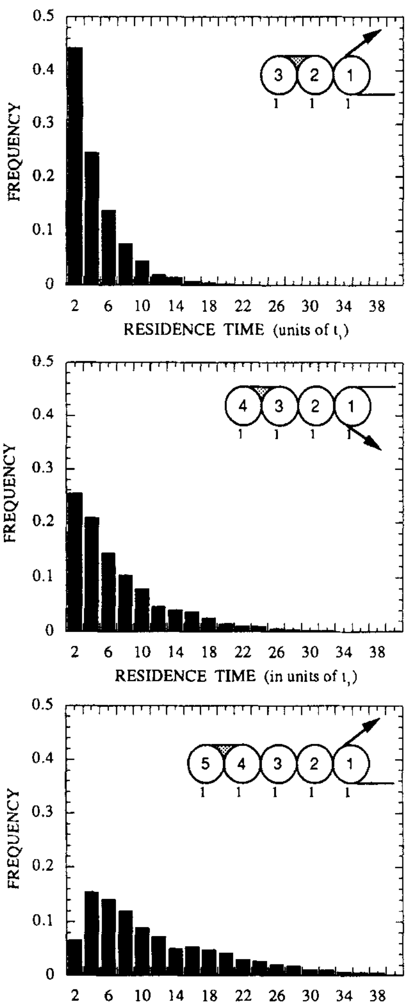

Figure 7. Residence time distributions of liquid coated from three-, four- and five-roll transfer coaters with equal roll speeds, equal roll radii, and straight-line roll arrangement.

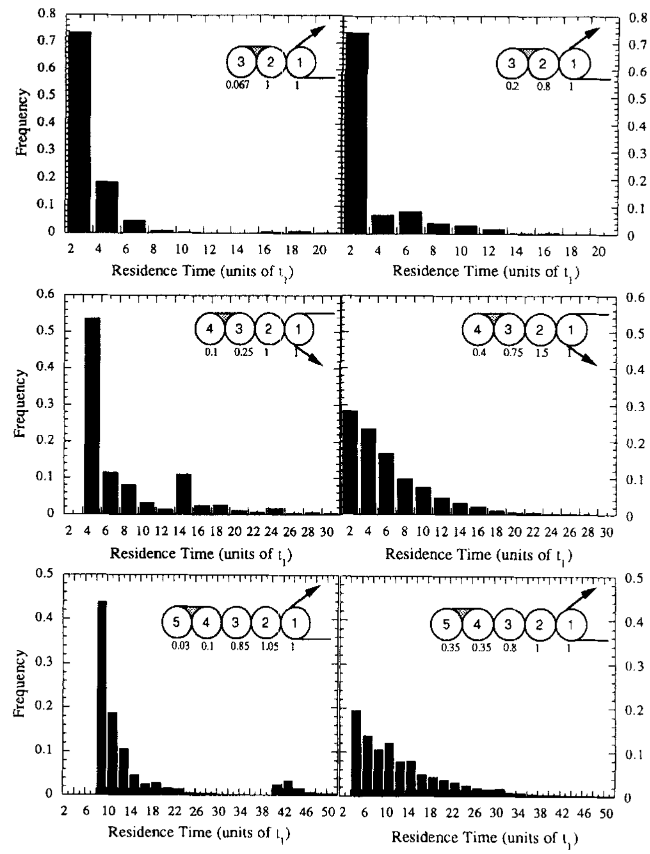

Figure 8. Residence time distributions of liquid coated from the six example transfer coaters listed in Table 1.

$\alpha = 1$; $\beta = 2/3$.

tial disadvantage, whatever advantages additional rolls bring. Values of the standard deviation σ are reported in Table 1.

Figure 8 shows distributions of residence times in liquid coated in the six examples listed in Table 1. Many of these coaters exhibit bimodal, trimodal, and multimodal distributions of residence time. Again, the distributions are wider for the coaters with more rolls.

Summary

The two limiting regimes of flow in the gaps, namely strictly steady, two-dimensional laminar flow without mixing, and somehow stirred flow with perfect mixing, presumably bracket what really goes on in multiple roll transfer coaters. Although these two regimes are easy to conceptualize and model, the one being a deterministic process and the other a stochastic, Markov process, getting at actual residence time distributions will take a lot more knowledge of mixing phenomena in gaps and their rolling banks than is available, so far as the authors are aware. Experimental observations of residence time distributions and phenomena that depend on them would be useful guides for further research in this area.

Dynamic Response

The foregoing analysis is of multiple roll coaters operating in steady state, which is desirable for successful coating. But the crux of precision roll coating is often roll run-out—departures from cylindrical symmetry and concentricity in bearings, alignments, and machining of shafts and roll surfaces. Run-out makes roll coating inherently unsteady. An important issue is the sensitivity of coating thickness uniformity to run-out. Also, operating conditions are often changed deliberately and abruptly, either slightly to meet target coating thickness, or sometimes more substantially because the target is changed. Then the time to reach the new (almost) steady state is an important issue. Thus there are needs for information about the dynamic response of multiple roll coating systems. This is especially true in ink distribution systems of printing presses where the ink is supplied to the inking rolls not continuously but periodically. The basic transient response of such systems was examined theoretically and experimentally by Mill (1961).

The dynamic response of a forward roll transfer coater to perturbation or adjustment of operating parameters can be divided into two parts: the time for liquid to travel from one gap to the next (transit time) and the time for the gap flows themselves to equilibrate in response to flow rate changes. Here only the effect of transit times is studied; the presumption is that at every instant the flow rate entering a gap flow zone equals the flow rate exiting that zone. This is sometimes referred to as a quasi-steady-state assumption.

Governing equations

The instantaneous mass balance at time t in the quasi-steady-state assumption at gap$_1$ is

$$V_2 h_2(t-t_2) = V_2 h'_2(t) + V_1 h_1(t). \tag{15}$$

Under unsteady conditions, the effective film thicknesses h_i and h'_i must be defined at some location on the roll because they may in fact vary around the rolls. The location chosen is the film-split, which is further taken to be the location of closest approach between the two rolls. Rewritten in terms of roll speed ratios, the mass balance at gap$_1$ becomes

$$V_{2,1} h_2(t-t_2) = V_{2,1} h'_2(t) + h_1(t). \tag{16}$$

The film-split relation at gap$_1$ is given by Eq. 2:

$$\frac{h'_2(t)}{h_1(t)} = \alpha_1 (V_{2,1})^{\beta_1}. \tag{17}$$

Combining Eqs. 16 and 17 gives an equation for $h_1(t)$ as a function of $h_2(t-t_2)$:

$$h_1(t) = \frac{V_{2,1} h_2(t-t_2)}{1+\alpha_1 (V_{2,1})^{1+\beta_1}}. \tag{18}$$

At intermediate gaps, there are two films entering and two leaving. The mass balance at an intermediate gap (gap$_i$) is

$$V_{i+1,i} h_{i+1}(t-t_{i+1}) + h'_i(t-t'_i) = V_{i+1,i} h'_{i+1}(t) + h_i(t) \qquad 2 \le i \le n-1. \tag{19}$$

The film-split ratio at gap$_i$ is given by Eq. 2:

$$\frac{h'_{i+1}(t)}{h_i(t)} = \alpha_i (V_{i+1,i})^{\beta_i} \qquad 2 \le i \le n-1. \tag{20}$$

Combining Eqs. 19 and 20 gives an equation for $h_i(t)$ in terms of $h_{i+1}(t-t_{i+1})$ and $h'_i(t-t'_i)$. From the film-split formula for gap$_{i-1}$ at time $t-t_i$, namely

$$\frac{h'_i(t-t'_i)}{h_{i-1}(t-t'_i)} = \alpha_{i-1} (V_{i,i-1})^{\beta_{i-1}} \qquad 2 \le i \le n-1, \tag{21}$$

an expression for $h_i(t)$ in terms of $h_{i-1}(t-t'_i)$ and $h_{i+1}(t-t_{i+1})$ follows:

$$h_i(t) = \frac{V_{i+1,i} h_{i+1}(t-t_{i+1}) + \alpha_{i-1}(V_{i,i-1})^{\beta_{i-1}} h_{i-1}(t-t'_i)}{1+\alpha_i (V_{i+1,i})^{1+\beta_i}} \qquad 2 \le i \le n-1. \tag{22}$$

With a pond feed system, as pictured in Figure 1, the feed film thickness $h_n(t)$ is a function of the gate roll gap $g_n(t)$:

$$h_n(t) = \frac{\lambda_n (1+V_{n+1,n})}{2\left[1+\alpha_n (V_{n+1,n})^{1+\beta_n}\right]} g_n(t). \tag{23}$$

In summary, there are n time-dependent equations for n film thicknesses (primary films) that are functions of the other primary film thicknesses at previous times. These constitute the set of dynamical equations that describe the response of a forward-roll, pond-fed transfer coater with $n+1$ rolls:

$$h_1(t) = f[h_2(t - t_2)]$$

$$h_2(t) = f[h_1(t - t'_2), h_3(t - t_3)]$$

$$h_3(t) = f[h_2(t - t'_3), h_4(t - t_4)]$$

$$\vdots$$

$$h_{n-1}(t) = f[h_{n-2}(t - t'_{n-1}), h_n(t - t_n)]$$

$$h_n(t) = f(g_n(t)).$$

The initial condition is that all film thicknesses are at their steady-state values corresponding to a feed gap width of g_n^0 and have been so in the preceding time $t < t^*$. At time t^*, the feed gap becomes a function of time, either executing a step change or oscillating around a mean value in a way that resembles the run-out of the two gate rolls.

The dynamical equations then need to be solved at discrete times: t^*, $t^* + \tau$, $t^* + 2\tau$, . . . , $t^* + n\tau$, where τ is a small fraction of the smallest transit time in the system. The film thicknesses were so computed and recorded at each time step. In this way, a history of each film thickness over times $t \geq t^*$ was compiled so that references to film thickness at past times, as required by the dynamical equations, could be made. The coated film thickness as a function of time was computed with various values of τ to determine the smallest τ necessary. It was found that when $\tau \leq 0.001 t_1$, the sensitivity of $h_1(t)$ to τ was totally negligible—it was less than computer precision of sixteen significant figures.

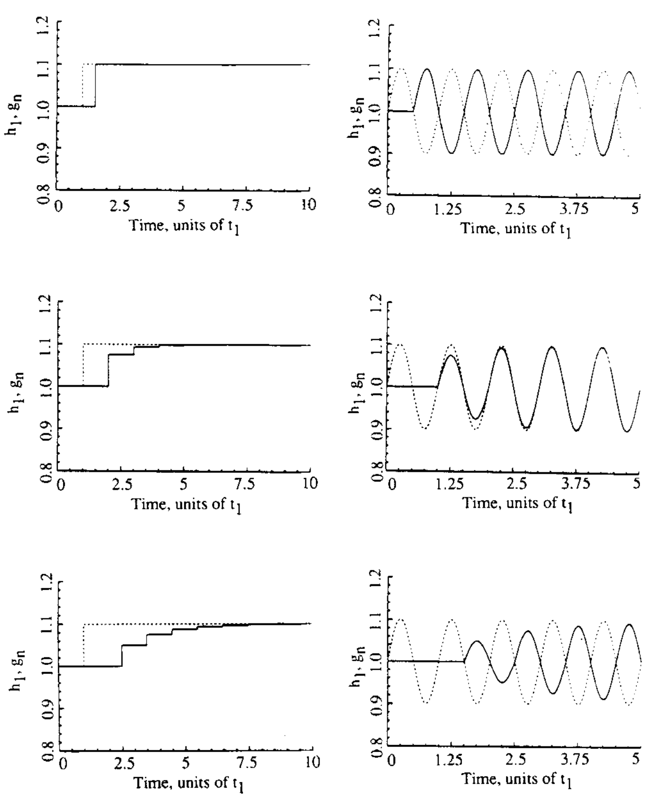

Figure 9. Dynamic response of coated film thickness to a step change in the feed gap (left) and to sinusoidal variation of the feed gap (right) by coaters with equal roll radii, equal roll speeds, and rolls arranged in a line: three-roll (top), four-roll (middle), and five-roll (bottom).
$---g_n(t);\ —h_1(t)$.

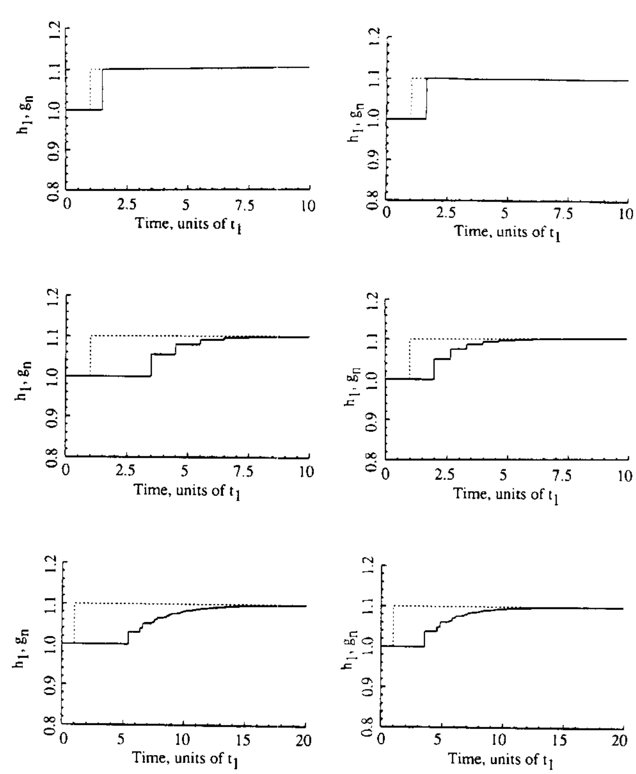

Figure 10. Dynamic response of coated film thickness to a step change in the feed gap by the six example transfer coaters listed in Table 1: Booth (top, left), Hebels (top, right), Satas (middle, right), Zahn (middle, right), Potjer (bottom, left), Weiss (bottom, right).
$---g_n(t);\ —h_1(t)$.

Step change in feed gap

The response to a step change of an otherwise steady feed gap,

$$g_n(t) = \begin{Bmatrix} g_n^0, & t < t^* \\ g_n^0(1 + \epsilon_n), & t \geq t^* \end{Bmatrix}, \tag{24}$$

where ϵ_n is the amplitude of the step change, was investigated.

Figures 9 and 10 show the responses of three-, four-, five-roll coaters to a step change in metering gap (at $t^* = t_1$). In these figures, the dotted line is the metering gap in units of the gap width at $t < t^*$; namely, $g_n(t)/g_n^0$; and the solid line is the coated film thickness in units of the coated film thickness at $t < t^*$; namely, $h_1(t)/h_1^0$. Both metering gap and film thickness can be plotted as functions of time in units of t_1. This dimensionless unit of time can be converted to length of coated web by multiplying by $2\pi r_1$.

Figure 9 shows the response to a step change when all roll speeds were set equal, as were all roll radii, and the rolls were arranged in a straight line (i.e., with their axes parallel

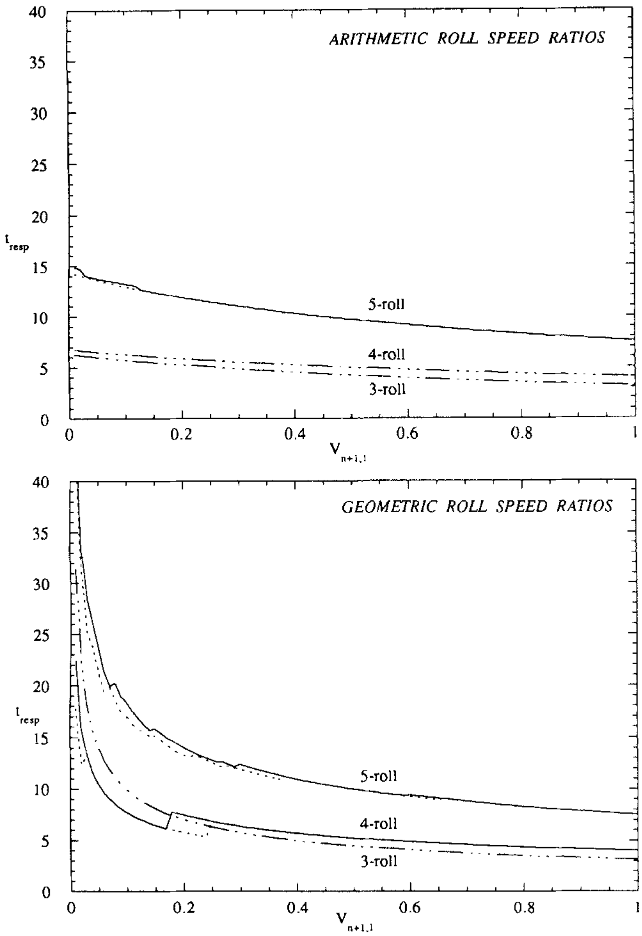

Figure 11. Response time of a coater to a step change in the feed gap, t_{resp}, depends on the number rolls, the roll speed ratios, the end roll speed, and the film-splitting exponent.

— $\beta = 1/3$; --- $\beta = 2/3$.

and coplanar). The response of the three-roll coater is a simple step change in the coated film thickness with a delay time equal to the transit time t_2. The four- and five-roll pond-fed transfer coaters do not show a simple step change in coated film thickness, but a gradual, stepwise approach to the new steady-state value. The reason is the succession of changes in the recycle streams, h'_2 in the four-roll system and h'_2 and h'_3 in the five-roll system, which are delayed by their motion back toward the feed gap. Nevertheless, the initial delay until any response in the coated film $h_1(t)$ can be observed is the minimum time for liquid to flow through the coater, t_{min}.

The elapsed time between when the gap width is changed and when the coated film thickness approaches to within some fraction of the new steady-state value, say 99%, is often called the response time of the system, t_{resp}. The response times of example transfer coaters are reported in Table 1. With three-roll transfer coaters, $t_{resp} = t_{min}$. With four- and five-roll coaters, t_{resp} is much greater than t_{min} and in most cases is three to five times t_{min}. The response time of three-, four-, and five-roll transfer coaters with intermediate roll speeds arranged in geometric and arithmetic progressions between the end-roll speed and line speed are shown in Figure 11. The results indicate that coaters with intermediate roll speeds in geometric progression have longer response times than do those with speeds in arithmetic progression. A lower film-splitting exponent results in slightly longer response times, again because proportionately more liquid flows into the time-delayed recycle streams.

Continuous perturbation of feed gap

When one or both of the gate rolls turns slightly eccentrically, or out-of-round, the metering gap is perturbed continuously. Out-of-roundness may be sinusoidal in shape around the circumference of the rolls, so that the gap varies sinusoidally:

$$g_n(t) = \begin{Bmatrix} g_n^0, & t < t^* \\ g_n^0[1 + \epsilon_n \sin(t/t_n) + \epsilon_{n+1} \sin(t/t_{n+1})] & t \geq t^* \end{Bmatrix}. \tag{25}$$

Here ϵ_n and ϵ_{n+1} are the amplitudes of the roll radius variation (roll run-out) of rolls n and $n+1$, respectively. This is the situation examined in what follows, excepting Figures 15 and 16.

Figures 9 and 12 show the response of coated film thickness $h_1(t)$, starting at $t = t^*$, to a single roll run-out ($\epsilon_n = 0$,

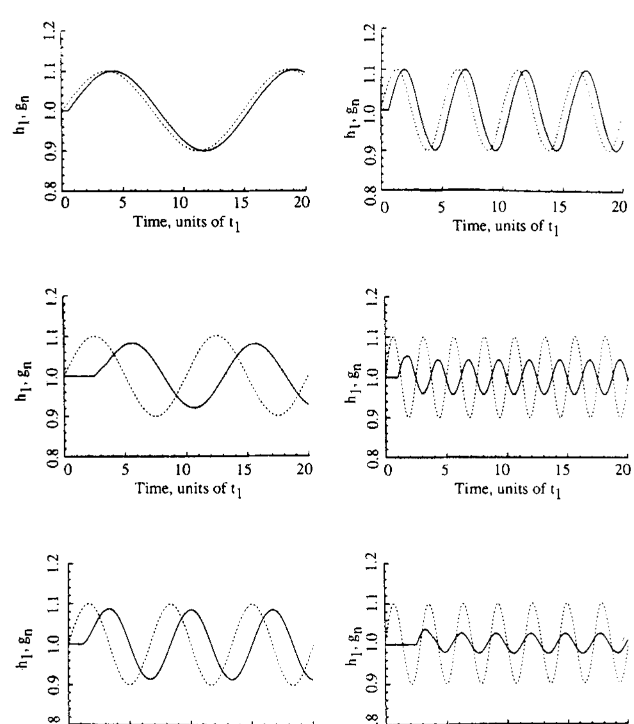

Figure 12. Dynamic response of coated film thickness to sinusoidal variation in the feed gap by the six coaters listed in Table 1: Booth (top left), Hebels (top right), Satas (middle left), Zahn (middle right), Potjer (bottom left), Weiss (bottom right).

--- $g_n(t)$; — $h_1(t)$.

$\epsilon_{n+1} = 0.1$) in the transfer coaters listed in Table 1. In general, the response is an initial delay time equal to $t_{\min}$ followed by a transient response that leads to steady oscillation in response to the gap variation. At steady oscillation, the coated film thickness varies sinusoidally with the same frequency in time as the feed gap runout and with an amplitude $|h_1(t) - h_1^0|_{\max}/h_1^0$ that is some fraction of the amplitude of the feed gap variation $|g_n(t) - g_n^0|_{\max}/g_n^0$. To quantify this effect, it is useful to define a runout damping factor $\mathfrak{D}$:

$$\mathfrak{D} \equiv \left(\frac{|h_1(t) - h_1^0|_{\max}/h_1^0}{|g_n(t) - g_n^0|_{\max}/g_n^0} \right)_{\text{steady oscillation}}, \tag{26}$$

where h_1^0 is the coated film thickness at steady state when the initial feed gap setting is g_n^0. (h_1^0 is also the average coated film thickness over time.) The displays of $h_1(t)$ vs. time in Figures 9 and 12 have not necessarily achieved steady oscillation. To determine $\mathfrak{D}$, the calculations were carried out until $|h_1(t) - h_1^0|_{\max}$ became independent of time so that steady oscillation, not the initial transient, was achieved.

Figure 9 shows the responses of three-, four-, and five-roll coaters in which the rolls turn at the same speed. In all three cases, the damping factor approaches unity as the performance of the coater achieves steady oscillation. The three-roll coater comes to such a state immediately after the delay time, $t_{\min}$ ($= t_2$). The more rolls used, the longer the delay time and the longer the initial transient before steady oscillation is achieved.

Figure 12 shows the response of the examples in Table 1 to a sinusoidal variation of the feed gap. Both of the three-roll coaters, in fact all three-roll coaters, have a damping factor of unity because the only recycle film h_2' flows back to the feed pond and thereby affects none of the gap flow rates. The four- and five-roll coaters exhibit significant damping, $\mathfrak{D} < 1$. For example, in the Zahn case ($\beta = 2/3$, $\mathfrak{D} = 0.25$), a variation of 10% of the feed gap results in a variation of only 2.5% in the coated film thickness. The frequency of the coated film thickness variation depends on the end-roll speed ratio $V_{n+1,1}$.

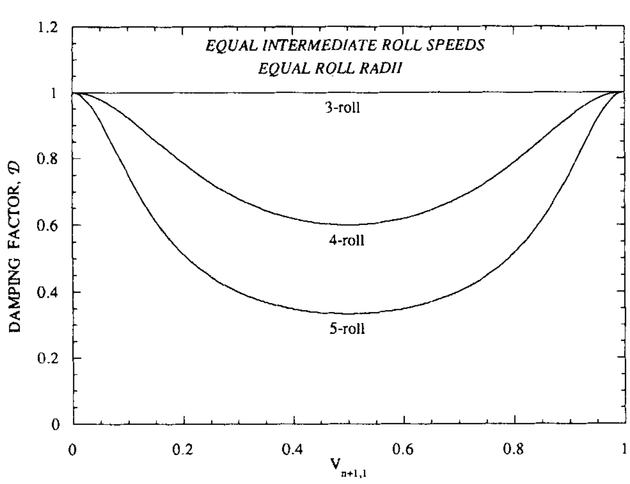

Figure 13. Damping factor $\mathfrak{D}$ is a strong function of the end-roll speed ratio $V_{n+1,1}$.

The intermediate roll speeds are equal to line speed and the roll radii are equal.

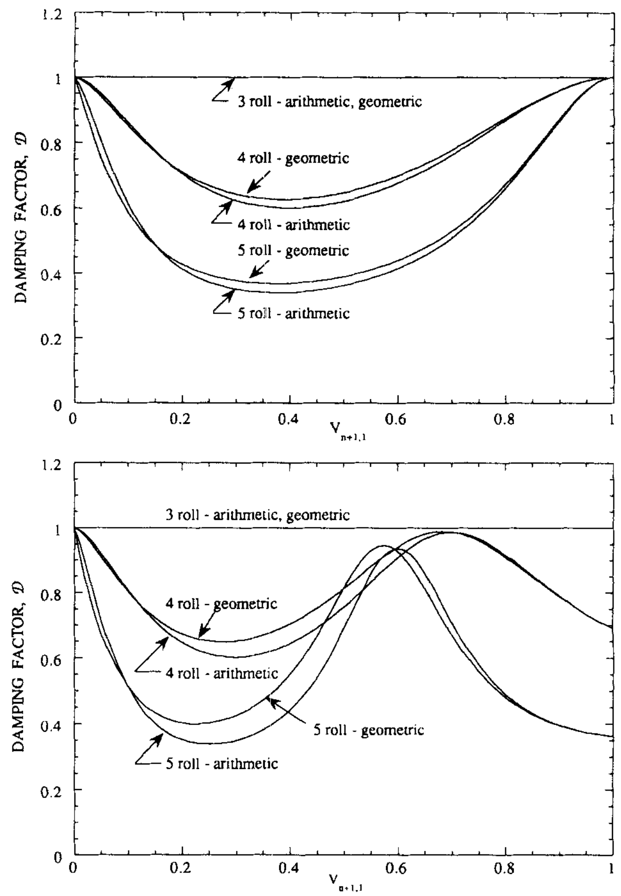

Figure 14. Damping factor $\mathfrak{D}$ is a strong function of the end-roll speed ratio $V_{n+1,1}$.

The intermediate roll speeds are arranged in geometric and arithmetic progressions between the end-roll speed and the line speed. In the upper graph, the roll radii are all equal; in the lower graph, the roll radius ratios are $r_{2,1} = 0.9$; $r_{3,1} = 0.8$; $r_{4,1} = 0.7$; $r_{5,1} = 0.6$; $\alpha = 1$; $\beta = 2/3$.

Figure 13 shows how $\mathfrak{D}$ depends on the end-roll speed when the intermediate roll speeds are equal and the roll radii are equal. The three-roll coater is unaffected by the end-roll speed because $\mathfrak{D} = 1$ always in three-roll coaters. Four- and five-roll coaters have the greatest damping at $V_{n+1,1} = 0.5$. The five-roll coater exhibits a greater damping, that is, smaller $\mathfrak{D}$, than the four-roll coater at comparable end-roll speed ratios.

The responses of coaters with intermediate roll speeds arranged in geometric progression and arithmetic progression between the end-roll speed and the line speed were also examined. Figure 14a shows the runout damping factor as a function of the end-roll speed $V_{n+1,1}$ when the roll radii are equal. Figure 14b shows the same when the roll radii decrease from the backing roll to the gate rolls: $r_{2,1} = 0.9$, $r_{3,1} = 0.8$, $r_{4,1} = 0.7$, $r_{5,1} = 0.6$. The damping factor seems to correlate with the ratio of the end-roll angular velocity to the backing roll angular velocity: the maximum damping, that is, minimum $\mathfrak{D}$, occurs when $\omega_{n+1}/\omega_1 \approx 0.5$.

Figure 15 shows the response of a four-roll coater (Zahn case from Table 1) to several different gap variations: a gap that varies due to runout of both gate rolls ($\epsilon_n = 0.1$, $\epsilon_{n+1} =$

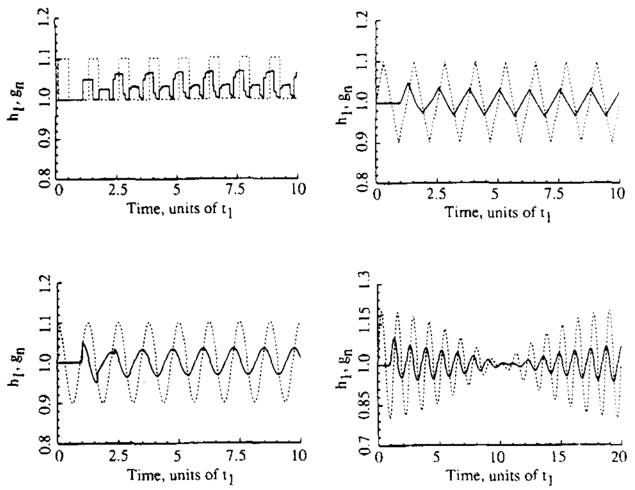

Figure 15. Dynamic response of coated film thickness to oscillatory variations of the feed gap by a four-roll coater (Zahn).

Square wave (upper left), saw-tooth (upper right), cosine (lower left), runout from both gate rolls— $\epsilon_{n+1} = 0.1$, $\epsilon_n = 0.1$ (lower right). $---g_n(t)$; $—h_1(t)$.

0.1) as a cosine wave, a square wave, and a saw-tooth pattern. Figure 16 shows the response of a five-roll coater (Potjer case from Table 1) to the same gap variation patterns.

Summary

The response time of multiple roll transfer coaters in coming to steady state after a step change in the feed gap was computed and was found to be most affected by the number of rolls and to a lesser extent by the roll speed ratios. The dimensionless response times t_{resp}/t_1 were found to range between 0.5 (three-roll, Booth case) and 20 (five-roll, Potjer case). This translates into about 0.2 s and about 8 s of response time, and about 0.3 m and 13 m of web distance, in those two cases, respectively, when the roll radii are 0.1 m and line speed is 100 m/min. The damping factor $\mathfrak{D}$ is less than or equal to one in every case examined. In all three-roll coaters and all coaters with equal roll speeds (roll speed ratios of unity), $\mathfrak{D}$ is equal to one: there is no damping. In coaters with more rolls, $\mathfrak{D}$ is smaller and depends strongly on the end-roll speed ratio, with a minimum value when $\omega_{n+1}/\omega_1 \approx 0.5$. The amount of damping by the transfer coater examples ranged from none ($\mathfrak{D} = 1$, three-roll coaters) to 75% ($\mathfrak{D} = 0.25$, five-roll coater, Zahn case).

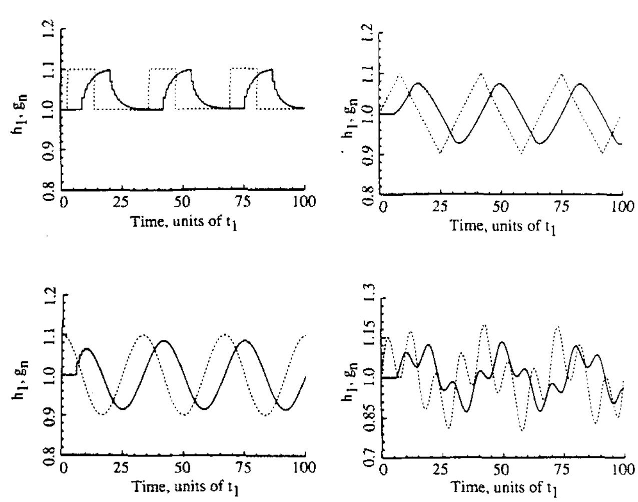

Figure 16. Dynamic response of coated film thickness to oscillatory variations of the feed gap by a five-roll coater (Potjer).

Square wave (upper left), saw-tooth (upper right), cosine (lower left), runout from both gate rolls— $\epsilon_{n+1} = 0.1$, $\epsilon_n = 0.1$ (lower right). $---g_n(t)$; $—h_1(t)$.

Conclusion

The residence time and the dynamic response are important as design criteria of multiple roll coaters. Experimental data about these things are not yet available. The foregoing analysis shows how models of multiple roll coaters based on simple mass balances and gap performance equations can be used to gain insight into the operation of multiple-roll systems. The models can also be used to interpret data as they become available.

The results make clear that the mean residence time and the residence time distribution depend on the number of rolls, the configuration of rolls, the roll speeds, the roll radii, and the degree of mixing in the individual gap flows. Two limits of gap flow mixing were considered: no mixing of liquid streams and perfect mixing of liquid streams; the latter led to a stochastic, Markov chain model of transfer roll coaters, from which predictions can be computed.

A quasi-steady-state model, accounting for transit time, was developed to simulate the dynamics of transfer coaters. It was used to show how transfer coaters respond to step changes in the feed gap and to continuous perturbation of the feed gap by roll run-out. The response times of examples of transfer coaters were computed and found to be affected most strongly by the number of rolls and to a lesser extent by the roll speed ratios. The response to continuous perturbation of the feed gap was characterized by a damping factor. It appears that coaters with four or more rolls damp somewhat the feed gap perturbation in the coated film thickness and the degree of damping depends most strongly on the number of rolls and the end-roll angular velocity.

Though they pertain to limiting cases, the results illustrate how fundamentals can be brought to bear on understanding, comparing, predicting, and ultimately designing multiple-roll systems. Experimental data on residence times and dynamic response are lacking, unfortunately. Just as no experiment can be totally accepted until confirmed by theory, so also no theoretical analysis can be totally accepted until confirmed by experiment. Documentation of how roll coating systems behave in practice is greatly needed.

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Literature Cited

Baily, N. T. J., *The Elements of Stochastic Processes with Applications to the Natural Sciences*, Wiley, New York (1964).

Benjamin, D. F., T. J. Anderson, and L. E. Scriven, "Multiple Roll Systems: Steady-State Operation," *AIChE J.*, **41**, 1045 (1995).

Benjamin, D. F., and L. E. Scriven, "Coaters Analyzed by Form and Function," *Ind. Coat. Res.*, **2**, 1 (1992).

Benjamin, D. F., M. S. Carvalho, T. J. Anderson, and L. E. Scriven, "Forward Roll Film-Splitting: Experiment and Theory," *Proc. TAPPI Coating Conf.*, San Diego, CA (May, 1994).

Benjamin, D. F., "Roll Coating Flows and Multiple Roll Systems," PhD Thesis, Univ. of Minnesota, Minneapolis (1994).

Benkreira, H., M. F. Edwards, and W. L. Wilkinson, "Roll Coating of Purely Viscous Liquids," *Chem. Eng. Sci.*, **36**, 429 (1981).

Booth, G. L., *Coating Equipment and Processes*, Lockwood, New York (1970).

Chung, K. L., *Markov Chains with Stationary Transition Probabilities* 2nd ed., Springer-Verlag, New York (1967).

Chung, K. L., *Elementary Probability Theory with Stochastic Processes*, 3rd ed., Springer-Verlag, New York (1979).

Coyle, D. J., C. W. Macosko, and L. E. Scriven, "Film-Splitting Flows in Forward Roll Coating," *J. Fluid Mech.*, **171**, 183 (1986).

Feller, W., *Introduction to Probability Theory and its Applications*, 3rd ed., Wiley, New York (1968).

Hammersly, J. M., and D. C. Handscomb, *Monte Carlo Methods*, Fletcher & Son Ltd., Norwich, U.K. (1964).

Hebels, A., Gorham International Conference on Release Papers and Films, Chicago (Aug., 1990).

Hintermair, J. C., and R. E. White, "The Splitting of a Water Film between Rotating Rolls," *TAPPI*, **48**(11), 617 (1965).

Howard, R. A., *Dynamic Programming and Markov Processes*, M.I.T. Press, Cambridge, MA (1960).

Mill, C. C., "An Experimental Test of a Theory of Ink Distribution," *Adv. in Printing Sci. and Technol.*, **1**, 183 (1961).

Morgan, B. J. T., *Elements of Monte Carlo Simulation*, Chapman & Hall, London (1984).

Potjer, B. R., personal communication, Avery, Leiden, The Netherlands (1992).

Satas, D., *Web Processing and Converting Technology and Equipment*, Van Nostrand Reinhold, New York (1984).

Savage, M. D., "Mathematical Models for Coating Processes," *J. Fluid Mech.*, **117**, 443 (1982).

Weiss, H. L., *Coating and Laminating Machines*, Converting Technology Co., Milwaukee, WI (1977).

Zahn, R. R., personal communication, GFG Corp., Milwaukee, WI (1993).

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